# Enumeration

## Counting Methods

Let $\#(n)$ denote the number of ways of doing $n$ things. Then,

$$\#(A \text{ and } B) = \#(A) \times \#(B)$$

$$\#(A \text{ or } B) = \#(A) + \#(B)$$

($n$ items, $r$ choices)
Ordered selection with repetition, $n^r$.

Ordered selection without repetition, $P(n,r) = \frac{n!}{(n-r)!}$.

Unordered selection without repetition, $C(n,r) = \frac{P(n,r)}{r!}$.

$|A \cup B| = |A| + |B| - |A \cap B|$

Ordered selection with repeition; WOOLLOOMOOLOO problem.

Unordered selection with repetition; dots and lines,

$$\binom{n+r-1}{n-1}$$

Pigeonhole principle. If you have n holes and more than n objects, then there must be at least 1 hole with more than 1 object.

# Recurrences

## Formal Languages

$\lambda$ represents the *empty word*. $w$ is just a variable (it is not part of the language)

## First Order Homogeneous Case

The recurrence,

$$a_n = ra_{n-1} \text{ with } a_0 = A$$

has solution

$$a_n = Ar^n.$$

## Second Order Recurrences

$$a_n + pa_{n-1} + qa_{n-2} = 0$$

has characteristic,

$$r^2 + pr + q = 0$$

If $\alpha$ and $\beta$ are the solutions to the characteristic equation, and if they are real and $\alpha \neq \beta$ then,

$$a_n = A\alpha^n + B\beta^n.$$

If $\alpha = \beta$ then,

$$a_n = A\alpha^n + Bn\beta^n.$$

## Guesses for a particular solution

| LHS | Guess |
|---|---|
| $3$ | c |
| $3n$ | $cn + d$ |
| $3 \times 2^n$ | $c2^n$ |
| $3n2^2$ | $(cn + d)2^n$ |
| $(-3)^n$ | $c(-3)^n$ |